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Kinetic Equation in the Kinetic Region of the Dilute and Nonuniform Electron Plasma

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Mori's scaling method is used to derive the kinetic equation for a dilute, nonuniform electron plasma in the kinetic region where the space–time cutoff (b, t_c) satisfies $\lambda_D \ll b \ll l_f$, $\tau_D \ll t_c \ll \tau_f$, with λ_D the Debye length, $\tau_D^{-1} = \omega_p$ the plasma frequency, and l_f and τ_f the mean free path and time, respectively. The kinetic equation takes account of the nonuniformity of the order of l_f and λ_D for the single- and the two-particle distribution function, respectively. Thus the Vlasov term associated with the two-particle distribution function is retained. This kinetic equation is deduced from the kinetic equation in the coherent region obtained by Morita, Mori, and Tokuyama, where the space–time cutoff of the coherent region satisfies $\lambda_D \gg b \gg r_0$, $\tau_D \gg t_c \gg \tau_0$, with r_0 the Landau length and τ_0 the corresponding time scale.

KEY WORDS: Coarse-graining in space and time; kinetic equation; BBGKY hierarchy; kinetic scaling; coherent scaling; nonuniform electron plasma.

1. INTRODUCTION

Recently, Mori's scaling method[1] for space–time coarse-graining has been used to clarify properties of fluctuations in μ-space[2,3] and the regime of validity of various kinetic equations in plasmas, such as the Balescu–Lenard–Guernsey equation.[4–6] The kinetic processes in a plasma are found to be characterized by two regions, the coherent and kinetic regions, where the kinetic region is a subregion of the coherent region. The divergence-free kinetic equation in these two regions has been systematically derived by applying Mori's scaling method for space–time coarse-graining to the BBGKY hierarchy equations and it has been demonstrated that the kinetic equation in the kinetic region can be reproduced from the kinetic equation in the coherent region by introducing further coarse-graining.

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Let us consider a classical electron gas with a small mean particle density c in a neutralizing, smeared-out background of positive charge with charge density ce, where e is the electronic charge. In this plasma the coherent region is defined by the space–time cutoff (b, t_c) which satisfies

$$\lambda_D \gg b \gg r_0, \qquad \tau_D \gg t_c \gg \tau_0 \tag{1.1}$$

The space–time cutoff $(b, t_c)$of the kinetic region satisfies

$$l_f \gg b \gg \lambda_D, \qquad \tau_f \gg t_c \gg \tau_D \tag{1.2}$$

Mori's scaling method leads to the following scalings of the characteristic quantities: for the coherent region

$$\lambda_D \to L\lambda_D, \qquad r_0 \to r_0, \qquad l_f \to L^2 l_f, \qquad c \to L^{-2}c \tag{1.3}$$

and for the kinetic region

$$l_f \to Ll_f, \qquad r_0 \to r_0, \qquad \lambda_D \to L^{1/2}\lambda_D, \qquad c \to L^{-1}c \tag{1.4}$$

The space–time (r_1, t) of the single-particle distribution function $f(1; t) = f(\mathbf{p}, \mathbf{r}_1; t)$ is scaled in both regions as

$$r_1 \to Lr_1, \qquad t \to Lt \tag{1.5}$$

In the work of Morita *et al.*[2] the collision term, which ensure the approach of $f(1; t)$ to the local Maxwell distribution function, consists of three terms:

$$C_1(f) = B_1(f) - L_1(f) + X_1(f) \tag{1.6}$$

where $B_1(f)$ is the Boltzmann collision term with the Coulomb potential and $L_1(f)$ is the Landau collision term. The third term $X_1(f)$ of Eq. (1.6) is a collision term first derived in Ref. 2.

In this paper, the kinetic equation in the nonuniform electron plasma is derived by expanding the collision term $C_1(f)$ in terms of the small parameter $1/L$. The derivation of this kinetic equation is given in Section 2. Section 3 is devoted to a discussion.

2. DERIVATION OF THE KINETIC EQUATION

Since the Boltzman and Landau collision terms are covariant in the kinetic scaling,[2] the expansion of the collision term $C_1(f)$ in terms of the small parameter $1/L$ comes only from the collision term $X_1(f)$, which is defined as

$$X_1(f) = \int d(2)\, \theta_{12} \int_0^\infty ds\, e^{s(M_1+M_2+M_3)}\theta_{12} f(1; t) f(2; t) \tag{2.1a}$$

with

$$\theta_{12} = \frac{\partial}{\partial \mathbf{r}_{21}} \frac{e^2}{|\mathbf{r}_{21}|} \cdot \left(\frac{\partial}{\partial \mathbf{p}_2} - \frac{\partial}{\partial \mathbf{p}_1} \right) = \theta_{21} \tag{2.1b}$$

$$M_1 = -\frac{\mathbf{p}_2 - \mathbf{p}_1}{m} \cdot \frac{\partial}{\partial \mathbf{r}_{21}} \tag{2.1c}$$

$$M_2 = c(1 + P_{12}) \int d(3)\, \theta_{13} P_{13} f(3; t) \tag{2.1d}$$

$$\begin{aligned} M_3 &= -\frac{\mathbf{p}_1}{m} \cdot \frac{\partial}{\partial \mathbf{r}_1} + c(1 + P_{12}) \int d(3)\, \theta_{13} f(3; t) \\ &= -\frac{\mathbf{p}_1}{m} \cdot \frac{\partial}{\partial \mathbf{r}_1} + (1 + P_{12}) \frac{\partial \Phi}{\partial \mathbf{r}_1} \cdot \frac{\partial}{\partial \mathbf{p}_1} \end{aligned} \tag{2.1e}$$

and

$$\Phi(\mathbf{r}_1; t) = c \int d(3) \frac{e^2}{|\mathbf{r}_{31}|} \exp\left(\mathbf{r}_{31} \cdot \frac{\partial}{\partial \mathbf{r}_1} \right) f(\mathbf{p}_3, \mathbf{r}_1; t) \tag{2.1f}$$

where $d(2) = d\mathbf{r}_2\, d\mathbf{p}_2$, $\mathbf{r}_{21} = \mathbf{r}_2 - \mathbf{r}_1$, and P_{ij} is the exchange operator between i and j. The term $X_1(f)$ is the collision term in the coherent region and scales as $X_1(f) \to L^0 X_1(f)$. In the coherent scaling of Eqs. (1.3) and (1.5), terms of $(M_1 + M_2 + M_3)$ scale as $M_1 + M_2 + M_3 \to L^{-1}(M_1 + M_2 + M_3)$. On the other hand, these terms scale differently in the kinetic scaling of Eqs. (1.4) and (1.5). The lowest order terms $(M_1 + M_2)$ scale as $(M_1 + M_2) \to L^{-1/2}(M_1 + M_2)$ and lead to the Balescu–Lenard–Guernsey collision term, as is shown in Ref. 2. The term M_3 scales as $M_3 \to L^{-1} M_3$. By keeping this term M_3, we can obtain the collision term in the nonuniform electron plasma.

We now define the function $K_q(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t)$ as

$$\begin{aligned} K_q(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t) = &\int d\mathbf{r}_{21} \exp(i\mathbf{q} \cdot \mathbf{r}_{21}) \\ &\times \int_0^\infty ds \exp[s(M_1 + M_2 + M_3)] \theta_{12} f(1; t) f(2; t) \end{aligned} \tag{2.2}$$

Making use of this function, we can write Eq. (2.1a) as

$$X_1(f) = -i \int d\mathbf{q}\, V_q \mathbf{q} \cdot \frac{\partial}{\partial \mathbf{p}_1} \int d\mathbf{p}_2 K_q(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t) \tag{2.3}$$

where $V_q = e^2/2\pi^2 q^2$. In the lowest order of the kinetic scaling of Eqs. (1.4) and (1.5), K_q and X_1 scale as $K_q \to LK_q$ and $X_1 \to L^0 X_1$, respectively. In order to obtain the expression for K_q, we use the following equations:

$$\begin{aligned}\int_0^\infty ds \exp[s(M_1 + M_2 + M_3) \\ &= \int_0^\infty ds \exp(sM_1) + \int_0^\infty du\, [\exp(uM_1)](M_2 + M_3) \\ &\times \int_0^\infty ds \exp[s(M_1 + M_2 + M_3)]\end{aligned} \tag{2.4}$$

$$\begin{aligned}\int d\mathbf{r}_{21} \exp(i\mathbf{q}\cdot\mathbf{r}_{21}) \int_0^\infty ds \exp\left(-s\mathbf{g}_{21}\cdot\frac{\partial}{\partial \mathbf{r}_{21}}\right) Q(\mathbf{r}_{21}) \\ &= \int_0^\infty ds \exp(i\mathbf{q}\cdot\mathbf{g}_{21}s) \int d\mathbf{r}_{21} \exp(i\mathbf{q}\cdot\mathbf{r}_{21})\, Q(\mathbf{r}_{21})\end{aligned} \tag{2.5}$$

where $\mathbf{g}_{21} = (\mathbf{p}_2 - \mathbf{p}_1)/m$ and $Q(\mathbf{r}_{21})$ is an arbitrary function of $\mathbf{r}_{21}$. Substituting Eq. (2.4) into Eq. (2.2) and making use of Eq. (2.5), we find

$$\begin{aligned}K_q(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t) \\ &= \frac{\sqrt{c}}{\pi} \frac{i}{\mathbf{q}\cdot(\mathbf{p}_2 - \mathbf{p}_1)/m + i\{(\mathbf{p}_1/m)\cdot\partial/\partial\mathbf{r}_1 - (\partial\Phi/\partial\mathbf{r}_1)\cdot(\partial/\partial\mathbf{p}_1 + \partial/\partial\mathbf{p}_2)\}} \\ &\times A(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t)\end{aligned} \tag{2.6}$$

$$A(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t) = A^{(0)}(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t) + A^{(1)}(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t) \tag{2.7}$$

where the first and the second terms in the curly brackets in the denominator of Eq. (2.6) are scaled by $L^{-1/2}$ and L^{-1}, respectively. The terms $A^{(0)}$ and $A^{(1)}$ scale as $A^{(0)} \to LA^{(0)}$ and $A^{(1)} \to L^{1/2}A^{(1)}$. The explicit form of $A^{(0)}$ and $A^{(1)}$ will be given in the subsequent analysis. We now follow Guernsey's analysis, using the following identity and the "barring" operation:

$$\frac{1}{X + Y} = \frac{1}{X} - \frac{1}{X} Y \frac{1}{X + Y} \tag{2.8}$$

$$\bar{f}(u, \mathbf{r}_1; t) = \int d\mathbf{p}_1\, \delta\left(u - \frac{\mathbf{q}\cdot\mathbf{p}_1}{m}\right) f(\mathbf{p}_1, \mathbf{r}_1; t) \tag{2.9}$$

Making use of Eqs. (2.8) and (2.9), we can calculate the function $G_q(\mathbf{p}_1, \mathbf{r}_1; t)$ from Eqs. (2.6) and (2.7) as follows:

$$G_q(\mathbf{p}_1, \mathbf{r}_1; t) = \int d\mathbf{p}_2\, K_q(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t) = G_q^{(0)}(\mathbf{p}_1, \mathbf{r}_1; t) + G_q^{(1)}(\mathbf{p}_1, \mathbf{r}_1; t) \tag{2.10}$$

where

$$G_q^{(0)}(\mathbf{p}_1, \mathbf{r}_1; t) = \frac{\sqrt{c}}{\pi} \int_{-\infty}^{\infty} dw \frac{i}{w-u} A^{(0)}(w, \mathbf{p}_1, \mathbf{r}_1; t)$$
$$= L_1(\mathbf{p}_1, \mathbf{r}_1; t) + iL_2(\mathbf{p}_1, \mathbf{r}_1; t) \tag{2.11a}$$

$$\bar{A}^{(0)}(w, \mathbf{p}_1, \mathbf{r}_1; t) = \int d\mathbf{p}_2\, \delta\left(w - \frac{\mathbf{q}\cdot\mathbf{p}_2}{m}\right) A^{(0)}(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t) \tag{2.11b}$$

$$L_1(\mathbf{p}_1, \mathbf{r}_1; t) = -\pi c D_q(\mathbf{p}_1, \mathbf{r}_1; t) H\left[\frac{\bar{q}_1(u)}{|\rho|^2}\right]$$
$$+ \frac{q_1(\mathbf{p}_1, \mathbf{r}_1; t)\rho_1 + q_2(\mathbf{p}_1, \mathbf{r}_1; t)\rho_2}{|\rho|^2} \tag{2.11c}$$

$$L_2(\mathbf{p}_1, \mathbf{r}_1; t) = \frac{\pi}{|\rho|^2}\{D_q(\mathbf{p}_1, \mathbf{r}_1; t)\bar{f}(u, \mathbf{r}_1; t) - f(1; t)\bar{D}_q(u, \mathbf{r}_1; t) \tag{2.11d}$$

$$D_q(\mathbf{p}_1, \mathbf{r}_1; t) = \frac{4\pi e^2}{q^2}\, \mathbf{q}\cdot\frac{\partial f(\mathbf{p}_1, \mathbf{r}_1; t)}{\partial \mathbf{p}_1} \tag{2.11e}$$

$$\rho = \rho_1 + i\rho_2; \qquad \rho_2(u, \mathbf{r}_1; t) = \pi c \bar{D}_q(u, \mathbf{r}_1; t),$$
$$\rho_1(u, \mathbf{r}_1; t) = 1 - H[\rho_2(u)] \tag{2.11f}$$

$$q_1(\mathbf{p}_1, \mathbf{r}_1; t) = -\pi D_q(\mathbf{p}_1, \mathbf{r}_1; t) H[\bar{f}(u)] + \frac{f(1; t)}{c} H[\rho_2(u)] \tag{2.11g}$$

$$q_2(\mathbf{p}_1, \mathbf{r}_1; t) = \pi D_q(\mathbf{p}_1, \mathbf{r}_1; t)\bar{f}(u, \mathbf{r}_1; t) - \frac{f(1; t)}{c}\rho_2 \tag{2.11h}$$

$$H[\rho_2(u)] = \frac{1}{\pi} P \int_{-\infty}^{\infty} dw \frac{\rho_2(w)}{w-u} \tag{2.11i}$$

$G_q^{(0)}$ scales as $G_q^{(0)} \to LG_q^{(0)}$ and the Balescu–Lenard–Guernsey collision term $BL_1(f)$ is found from Eqs. (2.3), (2.10), and (2.11) as

$$BL_1(f) = 8\pi^4 \int d\mathbf{q} \int d\mathbf{p}_2\, \mathbf{q}\cdot\frac{\partial}{\partial \mathbf{p}_1}\left|\frac{V_q}{\rho}\right|^2 \delta(\mathbf{q}\cdot\mathbf{g}_{12})\mathbf{q}\cdot\left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2}\right) f(1; t) f(\mathbf{p}_2, \mathbf{r}_1; t) \tag{2.12}$$

The collision term which takes account of the streaming and Vlasov terms of the two-particle correlation function can be derived from the next-order term

$G_q^{(1)}(\mathbf{p}_1, \mathbf{r}_1; t)$ defined as

$$
\begin{aligned}
G_q^{(1)}(\mathbf{p}_1, \mathbf{r}_1; t) = {} & \frac{\sqrt{c}}{\pi} \int_{-\infty}^{\infty} dw \frac{i}{w-u} A^{(1)}(w, \mathbf{p}_1, \mathbf{r}_1; t) \\
& + \frac{\sqrt{c}}{\pi} \frac{\mathbf{p}_1}{m} \cdot \frac{\partial}{\partial \mathbf{r}_1} \int_{-\infty}^{\infty} dw \frac{1}{w-u} \frac{\partial A^{(0)}(w, \mathbf{p}_1, \mathbf{r}_1; t)}{\partial w} \\
& - \frac{\sqrt{c}}{\pi} \frac{\partial \Phi}{\partial \mathbf{r}_1} \cdot \int_{-\infty}^{\infty} dw \frac{1}{w-u} \frac{\partial}{\partial w} \left\{ \frac{\partial A^{(0)}(w, \mathbf{p}_1, \mathbf{r}_1; t)}{\partial \mathbf{p}_1} \right. \\
& \left. + \int d\mathbf{p}_2\, \delta\left(w - \frac{\mathbf{q}\cdot\mathbf{p}_2}{m}\right) \frac{\partial A^{(0)}(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t)}{\partial \mathbf{p}_2} \right\}
\end{aligned}
\tag{2.13}
$$

where we have used the following equation to derive Eq. (2.13):

$$
\int_{-\infty}^{\infty} dw \frac{B(w)}{(w-u)^2} = \int_{-\infty}^{\infty} dw \frac{1}{w-u} \frac{\partial B(w)}{\partial w} \tag{2.14}
$$

with $B(w)$ an arbitrary function of w. The various quantities of Eq. (2.13) are defined as

$$
\begin{aligned}
& \frac{A^{(1)}(w, \mathbf{p}_1, \mathbf{r}_1; t)}{\pi\sqrt{c}} \\
& = \frac{1}{\pi\sqrt{c}} [A_R^{(1)}(w, \mathbf{p}_1, \mathbf{r}_1; t) + i A_I^{(1)}(w, \mathbf{p}_1, \mathbf{r}_1; t)] \\
& = L_1(\mathbf{p}_1, \mathbf{r}_1; t) \left\{ \frac{2\mathbf{q}}{q^2} \cdot \frac{\partial \bar{D}_q(w, \mathbf{r}_1; t)}{\partial \mathbf{r}_1} - \bar{D}_{\mathbf{r}_1}(w, \mathbf{r}_1; t) \right\} - \frac{\partial L_1(\mathbf{p}_1, \mathbf{r}_1; t)}{\partial \mathbf{q}} \\
& \quad \cdot \frac{\partial \bar{D}_q(w, \mathbf{r}_1; t)}{\partial \mathbf{r}_1} - \frac{2}{q^2} D_q(\mathbf{p}_1, \mathbf{r}_1; t) \mathbf{q} \cdot \frac{\partial \bar{L}_1(w, \mathbf{r}_1; t)}{\partial \mathbf{r}_1} \\
& \quad + D_{\mathbf{r}}(\mathbf{p}_1, \mathbf{r}_1; t) \bar{L}_1(w, \mathbf{r}; t)|_{\mathbf{r}=\mathbf{r}_1} + D_q(\mathbf{p}_1, \mathbf{r}_1; t) \left(\frac{\partial}{\partial \mathbf{r}_1} \cdot \frac{\partial}{\partial \mathbf{q}} \right) \bar{L}_1(w, \mathbf{r}_1; t) \\
& \quad + \frac{2f(1; t)}{cq^2} \mathbf{q} \cdot \frac{\partial \bar{D}_q(w, \mathbf{r}_1; t)}{\partial \mathbf{r}_1} - \frac{2}{cq^2} D_q(\mathbf{p}_1, \mathbf{r}_1; t) \mathbf{q} \cdot \frac{\partial \bar{f}(w, \mathbf{r}_1; t)}{\partial \mathbf{r}_1} \\
& \quad - \frac{f(1; t)}{c} \bar{D}\mathbf{r}_1(w, \mathbf{r}_1; t) + \frac{D_{\mathbf{r}}(\mathbf{p}_1, \mathbf{r}_1; t)}{c} f(w, \mathbf{r}; t)|_{\mathbf{r}=\mathbf{r}_1} \\
& \quad + i \left\{ L_2(\mathbf{p}_1, \mathbf{r}_1; t) \left[\frac{2}{q^2} \mathbf{q} \cdot \frac{\partial \bar{D}_q(w, \mathbf{r}_1; t)}{\partial \mathbf{r}_1} - \bar{D}_{\mathbf{r}_1}(w, \mathbf{r}_1; t) \right] \right. \\
& \quad \left. - \frac{\partial L_2(\mathbf{p}_1, \mathbf{r}_1; t)}{\partial \mathbf{q}} \cdot \frac{\partial \bar{D}_q(w, \mathbf{r}_1; t)}{\partial \mathbf{r}_1} \right\}
\end{aligned}
\tag{2.15a}
$$

$$\frac{A^{(0)}(w, \mathbf{p}_1, \mathbf{r}_1; t)}{\pi\sqrt{c}}$$

$$= \frac{1}{\pi\sqrt{c}}\{A_R^{(0)}(w, \mathbf{p}_1, \mathbf{r}_1; t) + iA_I^{(0)}(w, \mathbf{p}_1, \mathbf{r}_1; t)\}$$

$$= \frac{\pi}{|\rho|^2}\{2D_q(\mathbf{p}_1, \mathbf{r}_1; t)\bar{D}_q(w, \mathbf{r}_1; t)\bar{f}(u, \mathbf{r}_1; t)$$

$$- \bar{D}_q(u, \mathbf{r}_1; t)[\bar{D}_q(w, \mathbf{r}_1; t)f(1; t) + D_q(\mathbf{p}_1, \mathbf{r}_1; t)\bar{f}(w, \mathbf{r}_1; t)]\}$$

$$- i\frac{\rho_1}{|\rho|^2}\{\bar{D}_q(w, \mathbf{r}_1; t)f(1; t) - D_q(\mathbf{p}_1, \mathbf{r}_1; t)\bar{f}(w, \mathbf{r}_1, t)\} \tag{2.15b}$$

$$\frac{A^{(0)}(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1, t)}{\pi\sqrt{c}}$$

$$= \frac{1}{\pi\sqrt{c}}\{A_R^{(0)}(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t) + iA_I^{(0)}(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t)\}$$

$$= \frac{\pi}{|\rho|^2}\{2D_q(\mathbf{p}_1, \mathbf{r}_1; t)D_q(\mathbf{p}_2, \mathbf{r}_1; t)\bar{f}(u; t)$$

$$- \bar{D}_q(u, \mathbf{r}_1; t)[D_q(\mathbf{p}_2, \mathbf{r}_1; t)f(1; t) + D_q(\mathbf{p}_1, \mathbf{r}_1; t)f(\mathbf{p}_2, \mathbf{r}_1, t)]\}$$

$$- i\frac{\rho_1}{c|\rho|^2}\{D_q(\mathbf{p}_2, \mathbf{r}_1; t)f(1; t) - D_q(\mathbf{p}_1, \mathbf{r}_1; t)f(\mathbf{p}_2, \mathbf{r}_1; t)\} \tag{2.15c}$$

$$\bar{D}_{\mathbf{r}_1}(w, \mathbf{r}_1; t) = \frac{4\pi e^2}{q^2}\int d\mathbf{p}_2\, \delta\left(w - \frac{\mathbf{q}\cdot\mathbf{p}_2}{m}\right)\frac{\partial}{\partial \mathbf{r}_1}\cdot\frac{\partial f(\mathbf{p}_2, \mathbf{r}_1; t)}{\partial \mathbf{p}_2} \tag{2.15d}$$

$$D_{\mathbf{r}}(\mathbf{p}_1, \mathbf{r}_1; t) = \frac{4\pi e^2}{q^2}\frac{\partial f(1; t)}{\partial \mathbf{p}_1}\cdot\frac{\partial}{\partial \mathbf{r}} \tag{2.15e}$$

Making use of Eqs. (2.3), (2.10), (2.13), and (2.15), we find the collision term $Y_1(f)$, with which the kinetic equation for a dilute, nonuniform electron plasma can be written as

$$\left(\partial_t + \frac{\mathbf{p}_1}{m}\cdot\frac{\partial}{\partial \mathbf{r}_1}\right)f(1; t) = cJ_1(f) + c^{3/2}Y_1(f) \tag{2.16a}$$

with

$$J_1(f) = B_1(f) - L_1(f) + BL_1(f) \tag{2.16b}$$

$$\begin{aligned} Y_1(f) = \int d\mathbf{q}\, V_q \mathbf{q} \cdot \frac{\partial}{\partial \mathbf{p}_1} \Bigg(& H[\tilde{A}_R^{(1)}(u)] + \tilde{A}_I^{(1)}(u, \mathbf{p}_1, \mathbf{r}_1; t) \\ & + \frac{\mathbf{p}_1}{m} \cdot \frac{\partial}{\partial \mathbf{r}_1} \left\{ H\left[\frac{\partial \tilde{A}_I^{(0)}}{\partial w}(u)\right] - \left.\frac{\partial \tilde{A}_R^{(0)}(w, \mathbf{p}_1, \mathbf{r}_1; t)}{\partial w}\right|_{w=u} \right\} \\ & - \frac{\partial \Phi}{\partial \mathbf{r}_1} \cdot \left\{ H\left[\frac{\partial^2 \tilde{A}_I^{(0)}}{\partial \mathbf{p}_1\, \partial w}(u)\right] - \left.\frac{\partial^2 \tilde{A}_R^{(0)}(w, \mathbf{p}_1, \mathbf{r}_1; t)}{\partial \mathbf{p}_1\, \partial w}\right|_{w=u} \right\} \\ & - \frac{\partial \Phi}{\partial \mathbf{r}_1} \cdot \left\{ H\left[\frac{\partial \tilde{\mathbf{A}}_I^{(0)}}{\partial w}(u)\right] - \left.\frac{\partial \tilde{\mathbf{A}}_R^{(0)}(w, \mathbf{p}_1, \mathbf{r}_1; t)}{\partial w}\right|_{w=u} \right\} \Bigg) \end{aligned} \tag{2.16c}$$

where

$$\begin{aligned} & \tilde{\mathbf{A}}_R^{(0)}(w, \mathbf{p}_1, \mathbf{r}_1; t) + i\tilde{\mathbf{A}}_I^{(0)}(w, \mathbf{p}_1, \mathbf{r}_1; t) \\ & \quad = \int d\mathbf{p}_2\, \delta\left(w - \frac{\mathbf{q}\cdot\mathbf{p}_2}{m}\right) \left[\frac{\partial A_R^{(0)}(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t)}{\partial \mathbf{p}_2} + i\, \frac{\partial A_I(\mathbf{p}_2, \mathbf{p}_1, \mathbf{r}_1; t)}{\partial \mathbf{p}_2}\right] \end{aligned} \tag{2.16d}$$

The explicit expressions for $J_1(f)$ can be seen in Refs. 2 and 3. Both of the terms $J_1(f)$ and $Y_1(f)$ scale as $J_1(f) \to L^0 J_1(f)$ and $Y_1(f) \to L^0 Y_1(f)$. Thus the second term on the r.h.s. of Eq. (2.16a) cannot be derived by the density expansion. The first two terms of Eq. (2.16c) represent the correction of the shielding effect associated with the nonuniformity of the single-particle distribution function. The terms $(\mathbf{p}_1/m)\cdot(\partial/\partial\mathbf{r}_1)\{\cdots\}$ and $(\partial\Phi/\partial\mathbf{r}_1)\cdot\{\cdots\}$ represent the contributions from the streaming and the Vlasov terms of the two-particle distribution function, respectively.

3. DISCUSSION

In the conventional theory of deriving the Balescu–Lenard–Guernsey equation,[4–6] the following assumptions (or equivalent ones) are usually introduced[7]: (a) the single-particle distribution function is independent of position, (b) the relaxation time of the two-particle correlation function is much shorter than that of the single-particle distribution function.

In attempts to derive the kinetic equation for a nonuniform electron plasma along the lines of conventional theory,[7–10] the following condition is assumed:

$$f(\mathbf{p}, \mathbf{r}, t) = f_0(\mathbf{p}, t) + f_1(\mathbf{p}, \mathbf{r}, t); \qquad f_0 \gg f_1 \tag{3.1}$$

and the linearized kinetic equation for $f_1(\mathbf{p}, \mathbf{r}, t)$ is derived and discussed qualitatively. The main idea of these theories can be understood as follows. In a neutral gas, the relaxation time of the single-particle distribution function to the Maxwellian is the mean free time, while the relaxation time of a nonuniform gas to a uniform state is the macroscopic time scale. Therefore the relaxation time of the momentum and real space are completely different in the neutral gas. Considering this fact, the single-particle distribution function in a plasma is divided into two terms as shown in Eq. (3.1). This assumption, however, is not applicable to a plasma due to the long-range nature of the Coulomb force.

In the conventional approach to obtaining the kinetic equation for a nonuniform electron plasma, it is extremely difficult to derive it without making some assumptions. In the author's opinion, these assumptions have been motivated by a desire to simplify the problem rather than by the physics involved. Thus, to tackle such problems, a new method is needed. Mori's scaling method is such a method. In fact, we did not have to make the assumptions of the conventional theory in order to derive the kinetic equation for the dilute, nonuniform electron plasma.

In Mori's scaling method, the kinetic equation in the kinetic region includes the streaming term since the single-particle distribution function includes the nonuniformity of the order of the mean free path. The Vlasov term vanishes because the force range of the Coulomb force is of the order of λ_D, so that this term does not appear in the nonuniformity of the order of l_f. Both terms, i.e., the streaming and Vlasov terms, appear in the two-particle correlation function as a next order in the expansion of the small parameter $1/L$. The Vlasov term appears since the characteristic length between the two particles is taken to be of the order of λ_D. Thus in the lowest order we reproduce the Balescu–Lenard–Guernsey equation. By taking account of the next-order term in the expansion of the small parameter $1/L$, we derive the kinetic equation (2.16) for the dilute, nonuniform electron plasma, where the characteristic length of the nonuniformity of the single-particle distribution function is taken as l_f, while the characteristic length between the two particles for the two-particle distribution function is λ_D. These two conditions for the single- and two-particle distribution functions can be realized when a plasma without a static magnetic field is produced in the laboratory.

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